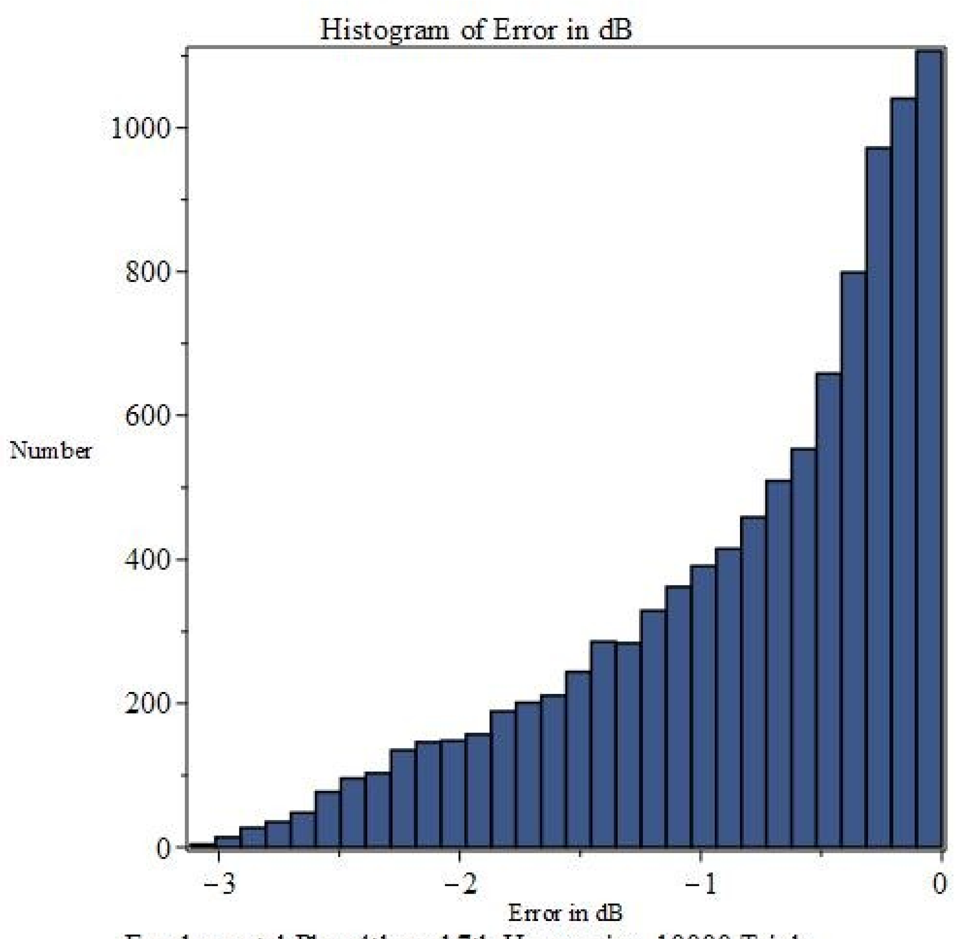
Revised Edition, 22 July 2020

Hi all,

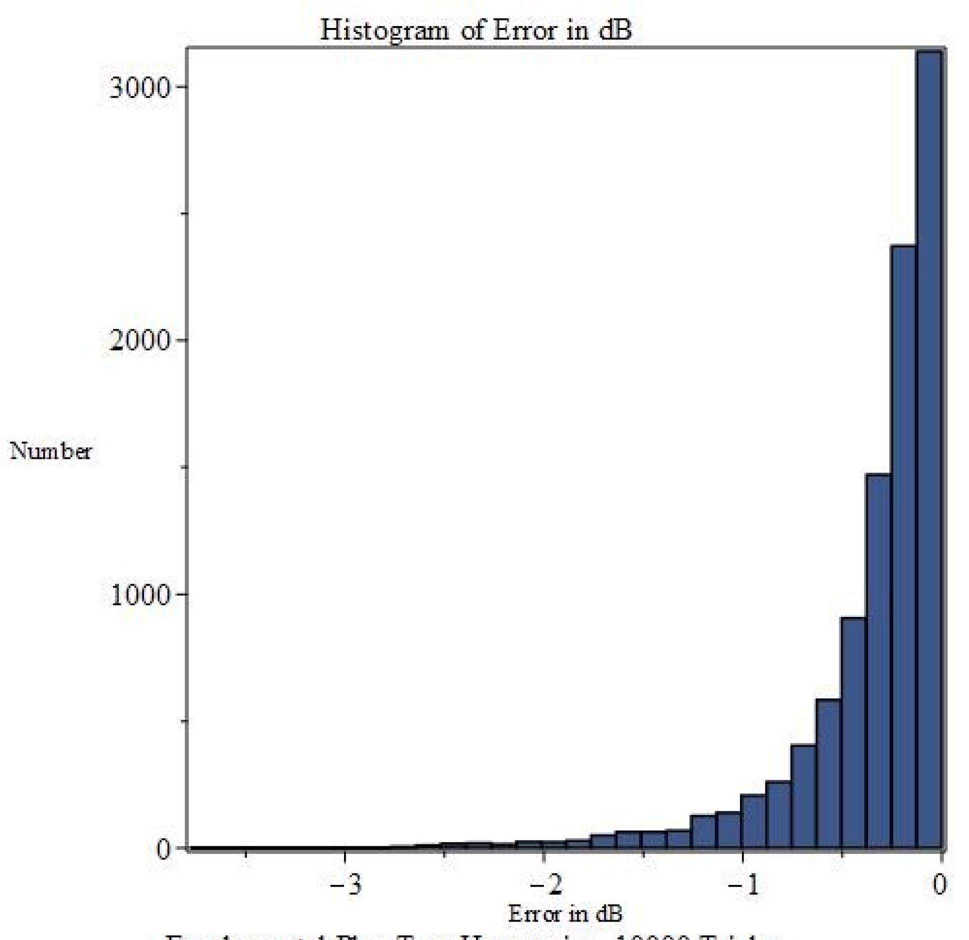
I decided to statistically evaluate the sums of randomly selected harmonics with random phases and amplitudes both to see how their maxima actually compared to the sums of the amplitudes and to learn a bit about how to use the new math package I recently obtained to replace some older packages.

First I simulated 10,000 variations of the problem described by the OP, namely a fundamental and two harmonics, specifically 4x and 7x. The phases were sampled from a uniform distribution between 0 and 2*Pi. The amplitudes were sampled from a uniform distribution between 0.1 and 1.0. The values for 1,000 evenly spaced points were calculated for each waveform before summation. The maximum sum was extracted and compared to the sum of the amplitudes. The "error" was calculated in dB. The result was that the median "error" was about -0.61 dB.



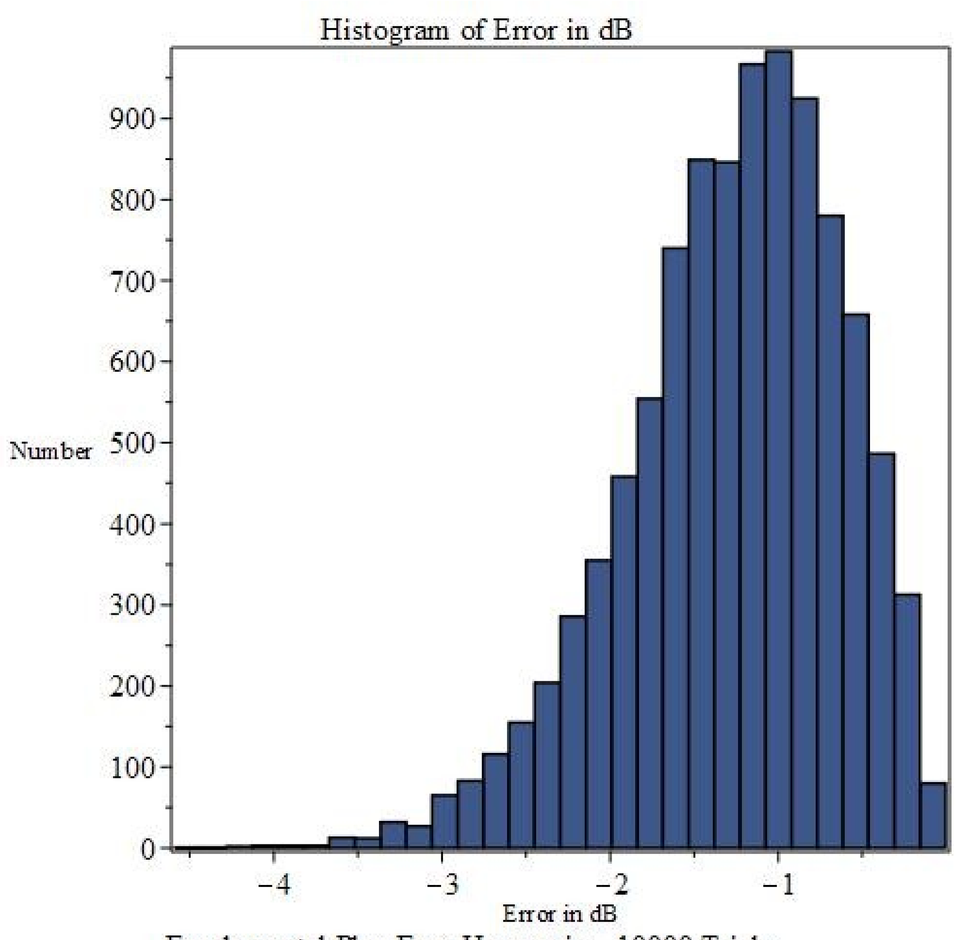
Fundamental Plus 4th and 7th Harmonics, 10000 Trials

Next I simulated 10,000 variations of a modified version of the problem above to determine the effect of allowing two harmonics to be chosen randomly from a limited set of values from 2x to 32x. (The OP posted 2x to 128x but did not specify the range of the number of harmonics, nor how it might vary.) The values for the harmonics for each trial were sampled from a uniform distribution from 2 to 32 inclusive. The result was a median "error" of -0.22 dB. The tail was very skinny and long. For this case, for 50% of the time there will be very good agreement between the sums of the amplitudes and the maximums of the sums of the harmonics.

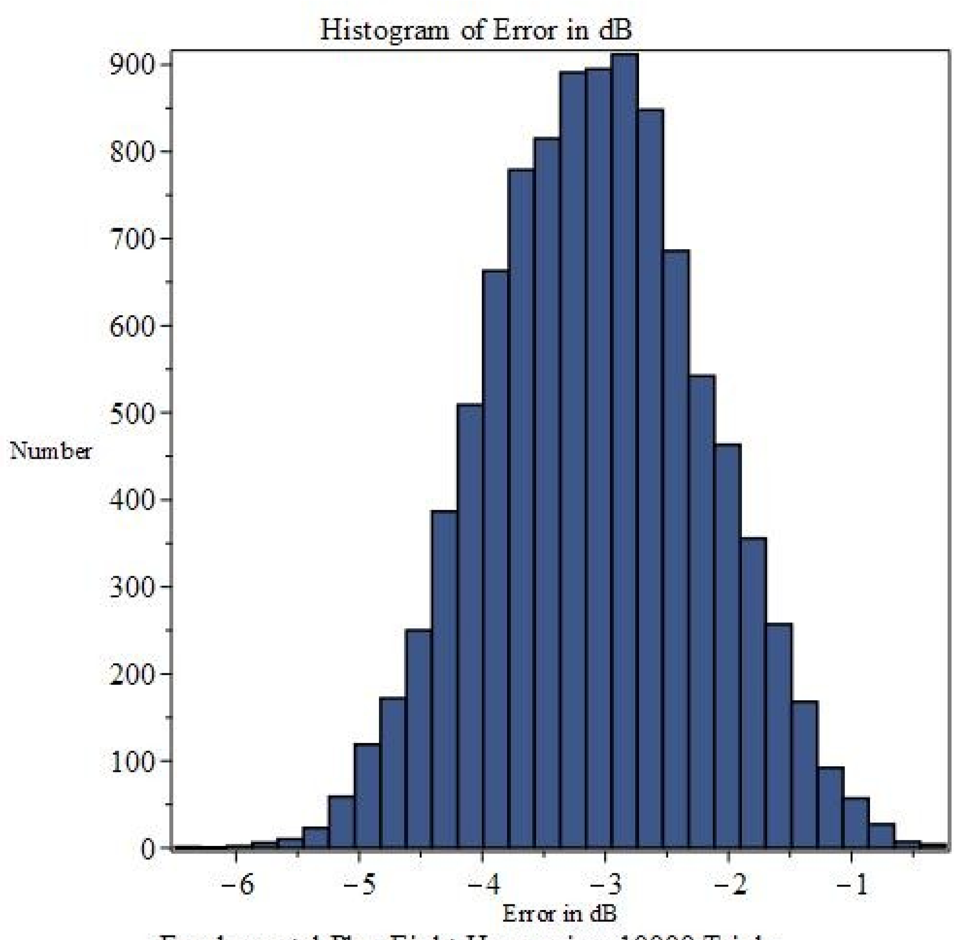


Fundamental Plus Two Harmonics, 10000 Trials

I repeated the same type of simulation with the fundamental plus four harmonics from 2x to 32x, with the result that the median "error" was -1.2 dB, then fundamental plus eight harmonics with the result of -3.1 dB.

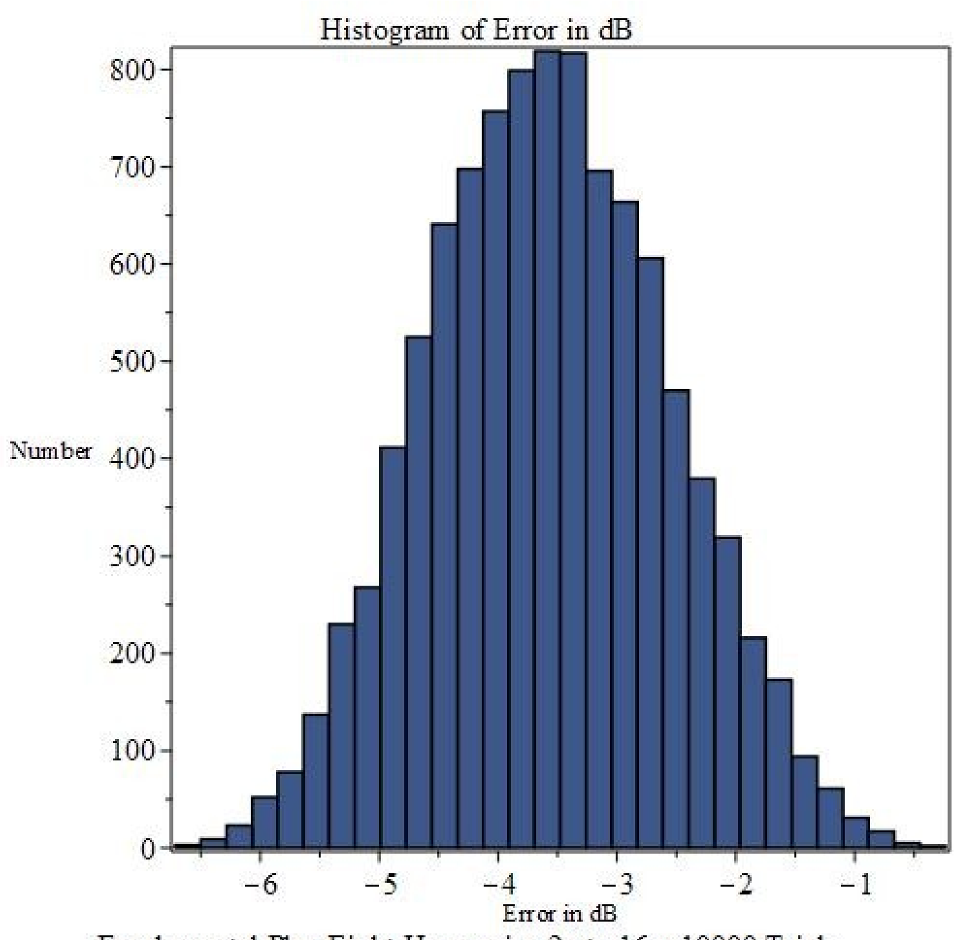


Fundamental Plus Four Harmonics, 10000 Trials

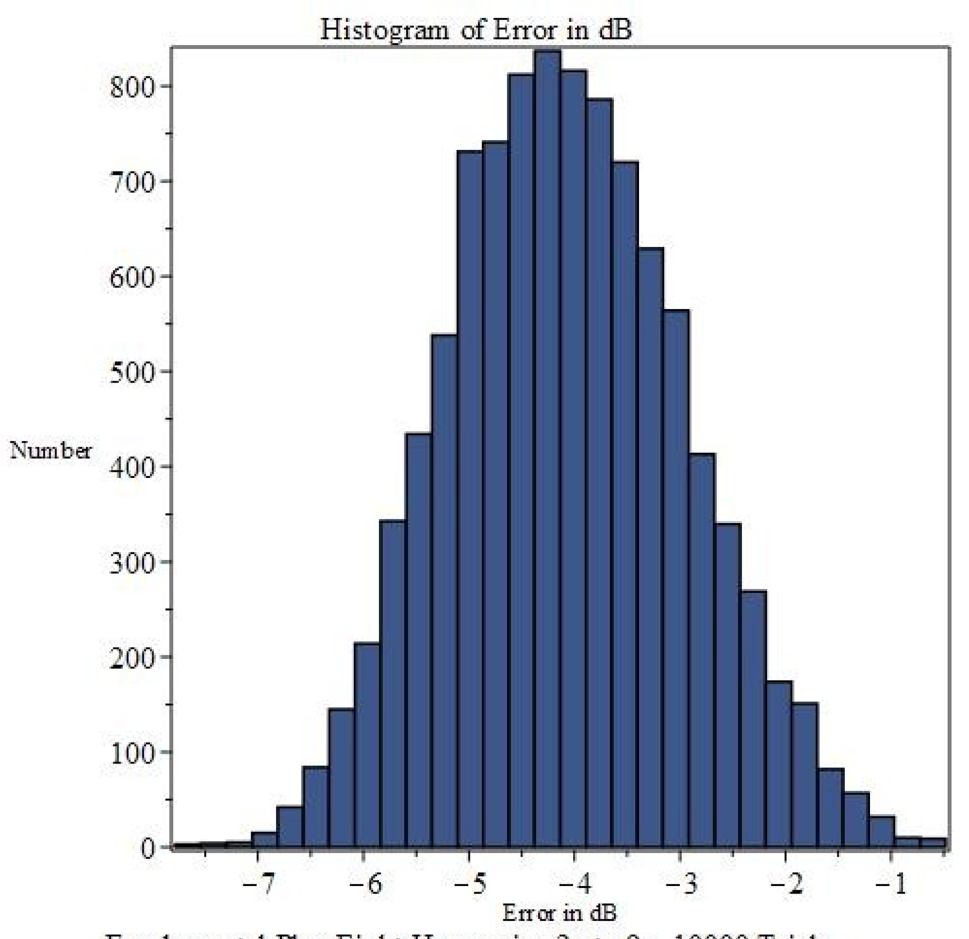


Fundamental Plus Eight Harmonics, 10000 Trials

To determine what a more dense set of harmonics might produce, I then limited the range of harmonics from 2x to 16x and repeated the calculations for fundamental plus eight harmonics with the result that the median "error" was -3.6 dB. The next result was for 2x to 9x for a full set of harmonics in that range with randomly varying phase and amplitude and was -4.1 dB for the median "error."

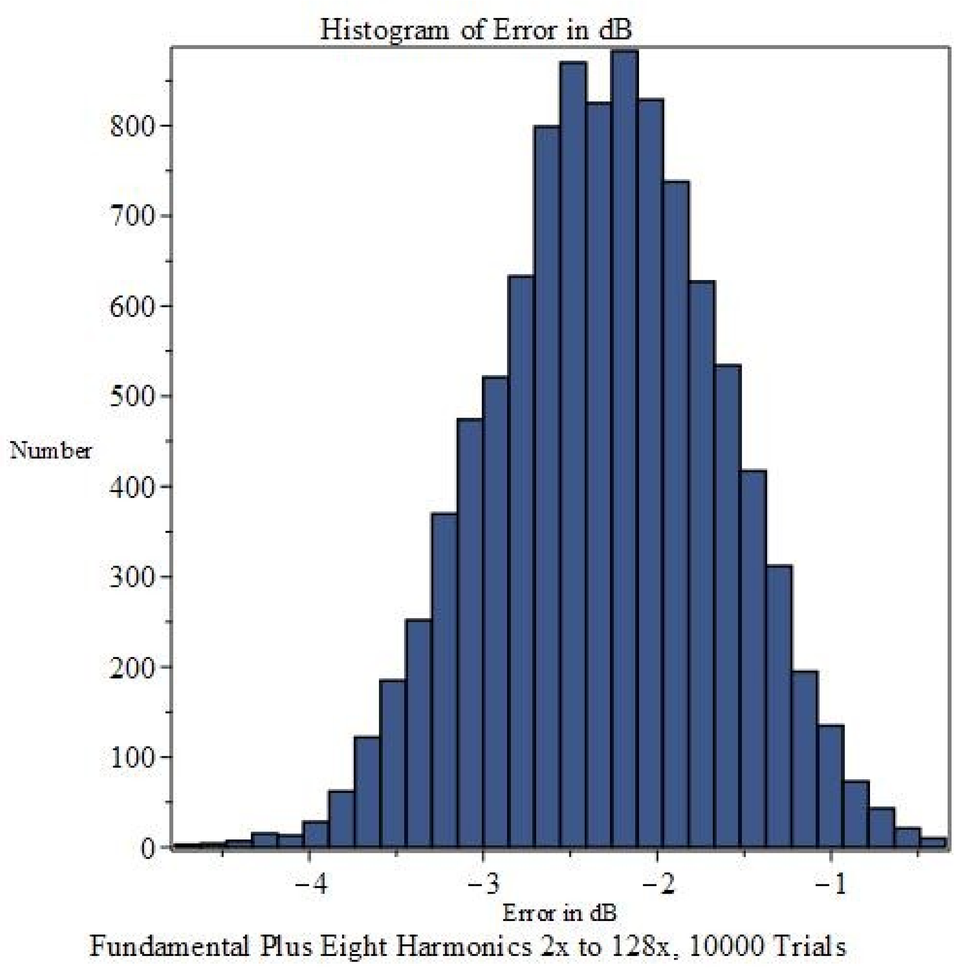


Fundamental Plus Eight Harmonics 2x to 16x, 10000 Trials



Fundamental Plus Eight Harmonics 2x to 9x, 10000 Trials

Out of curiosity about a range of harmonics for 2x to 128x, I looked at the fundamental plus eight harmonics with the result of -2.3 dB for both the median and the average "error."



Fundamental Plus Eight Harmonics 2x to 128x, 10000 Trials

These results seem to be at odds with what I understood some posters to have described, so perhaps there is some difference between the methods I used and what they understood to be the case, or perhaps there is even some mistake I've made with this new math package. What I see here is questionable or somewhat reasonable agreement to excellent agreement with occasionally bad agreement rather than the opposite. I would be interested in knowing if anyone else has obtained quantitative results.

Regards,
Dave Clark